Wavelet-based multiscale atlas estimation

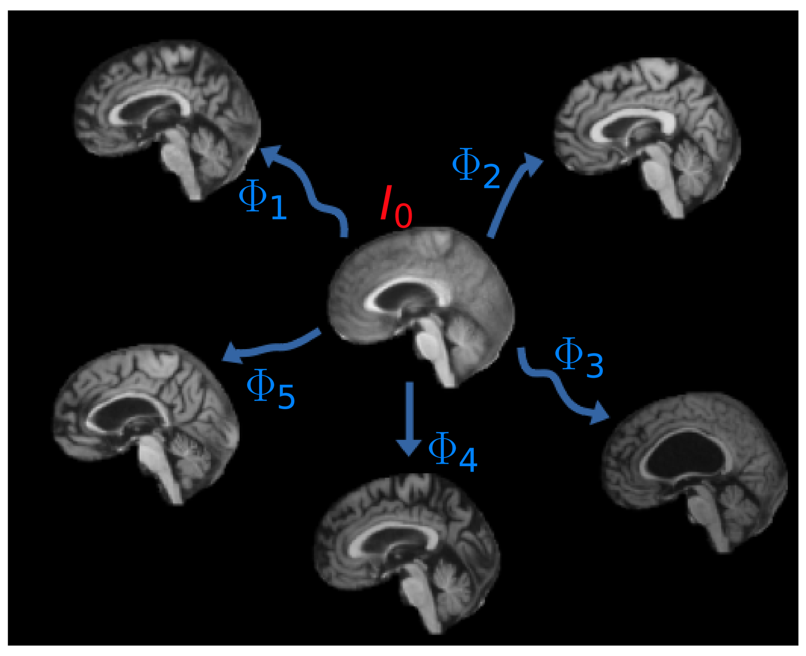
Fleur Gaudfernau[1], Stéphanie Allassonière[1] and Erwan Le Pennec[2]

[1] CRC, Université de Paris, INRIA EPI HeKa, INSERM UMR 1138, Sorbonne Université [2] CMAP, Ecole polytechnique, Institut Polytechnique de Paris

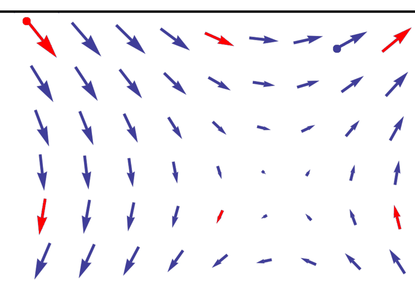
Introduction

Atlas estimation: given a dataset of shapes $[I_1, ..., I_n]$ that are instances of the same anatomical object, we seek to estimate:

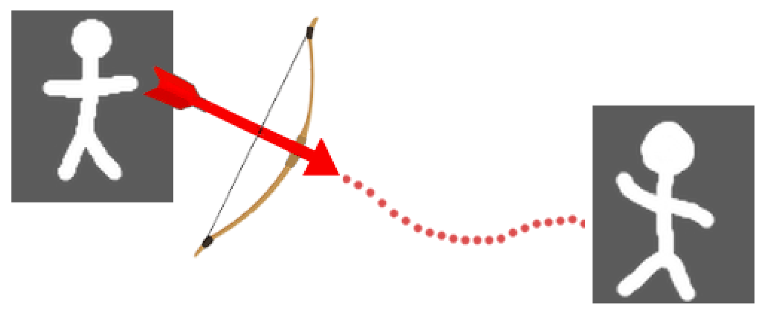
- a template image I_0 (average anatomy)
- n template-to-subject deformations Φ_i (variance) s.t. $I_i = I_0 \circ \Phi_i^{-1} + \epsilon_i$ (with ϵ_i an additive random white noise)



The LDDMM framework:



- The Φ_i are constructed by integrating time-dependant velocity fields $v_{t,i}$
- These vector fields belong to a RKHS defined by a regularizing kernel K_g
- Discrete parametrization of the vector fields: $v_0(x) = \sum_{k=1}^{k_g} \underbrace{K_g}_{\text{Regularizing kernel of width } \sigma_g}(x, \overbrace{c_{k,0}}^{\text{control points}}) \underbrace{\alpha_{k,0}}_{\text{momentum vectors}}$
- Geodesic shooting: to compute shape transformations, we only optimize the **initial** vector field v_0 (i.e. the vectors $(\alpha_0)_k$) that define a geodesic path between a source shape and a target shape.

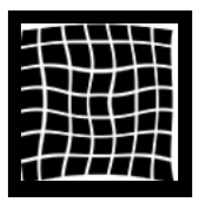


Geodesic shooting between a source image (left) and a target image (right).

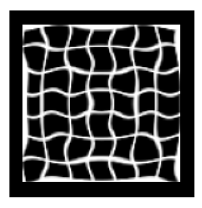
- Cost function: $E(I_0, (\alpha_{0,i})_i) = \sum_{i=1}^{N} \underbrace{\frac{1}{2\sigma^2} \|I_i - I_0 \circ \Phi_{1,i}^{-1}\|^2}_{\text{attachment}} + \underbrace{\|v_{0,i}\|_V^2}_{\text{regularity}}$
- Optimization through gradient descent $\longrightarrow$ efficient computation of the gradients because the v_0 are linear combination of kernels of the RKHS

Issues:

1. **Deformations $v_{0,i}$ are constrained to a single scale by the kernel width σ_g**
2. **Risk of trapping the optimization in an unrealistic local minima**



Deformation grid generated by a large kernel



Deformation grid generated by a small kernel

Core idea

Decompose the subjects' velocity fields $v_{0,i}$ in a new basis

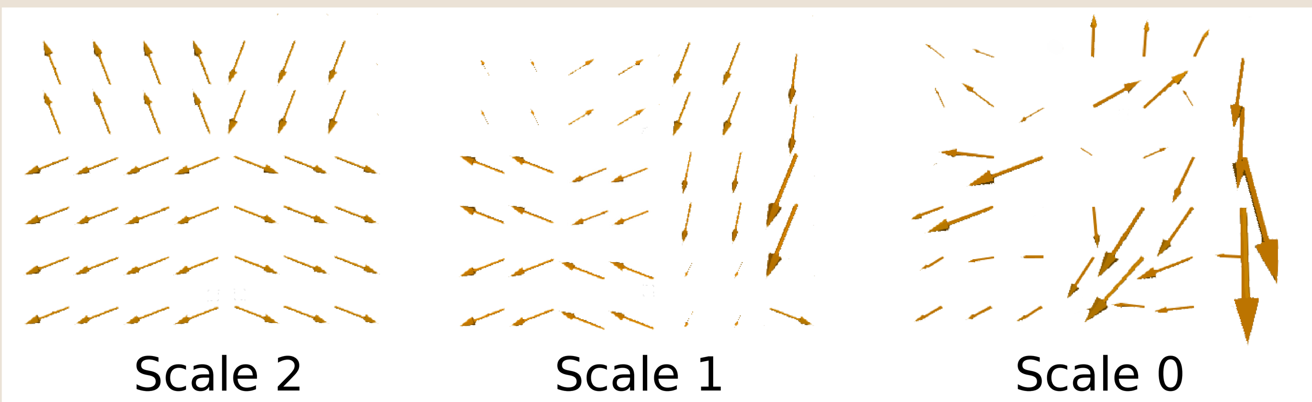
- Original coordinates α_i in the RKHS basis: $v_{0,i}(x) = \sum_k K_g(x, c_k)\alpha_{k,i}$ where $K_g(., c_k)$ is a function of scale s and location k

$\Longrightarrow$ Spatial regularization through the L^2 norm $\|v_{0,i}\|_V^2$

- New coordinates β_i in a wavelet basis: $v_{0,i}(x) = \sum_s \sum_k \sum_o \phi^o_{s,k}(x)\beta^o_{s,k,i}$ with ϕ a function of scale s, location k and orientation o

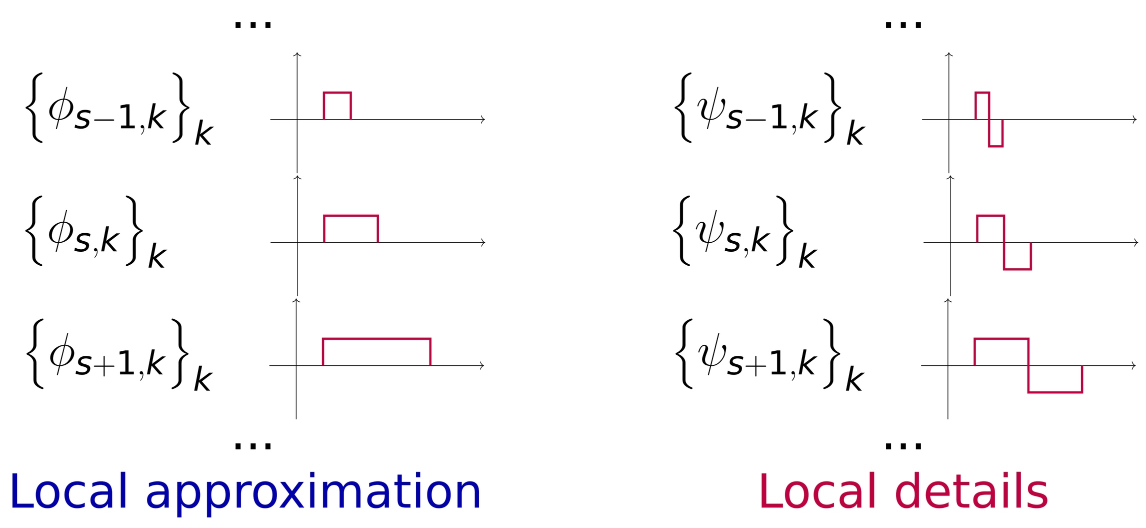
$\Longrightarrow$ We add a layer of spatial regularization

We set to zero some detail coefficients in β_i to smooth the vector fields. These constraints are progressively relaxed in a coarse-to-fine fashion.



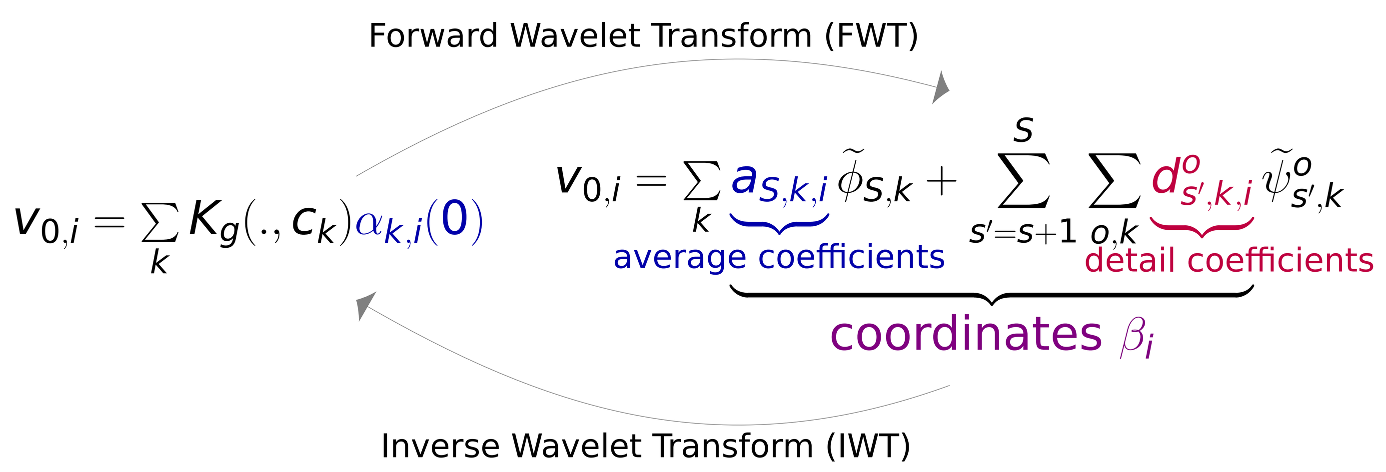
The Haar wavelet transform

- Multiscale decomposition of signals
- Rely on a collection of nested spaces



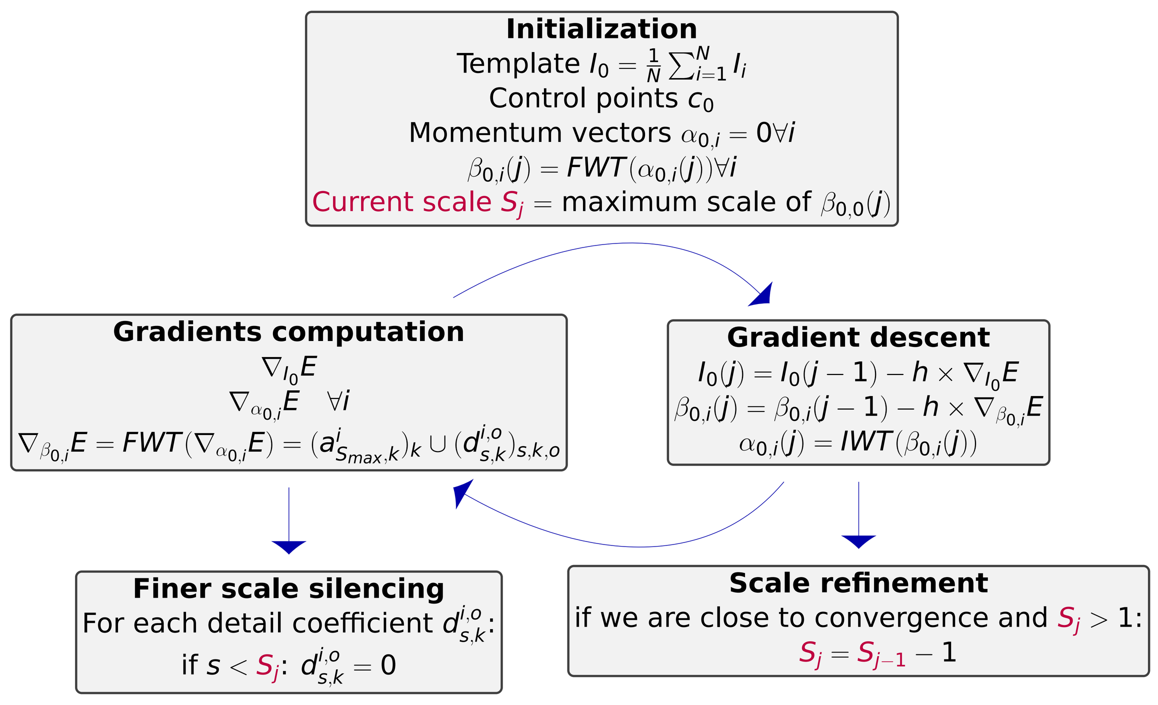
with ϕ: scaling function, ψ: wavelet function, s: scale, k: location

Reparametrization of v_0

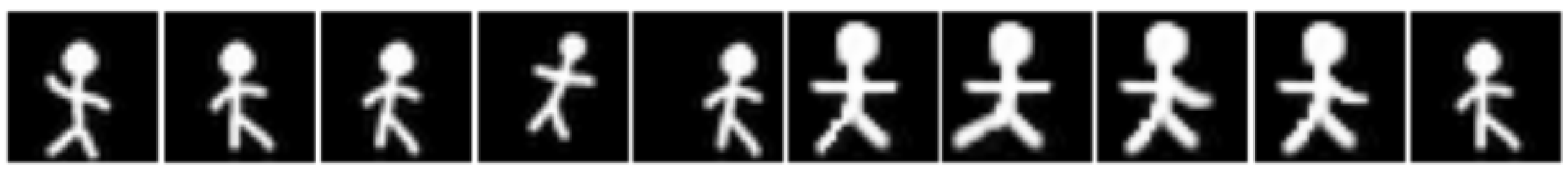


Even if we define $v_{0,i}$ through the wavelet coefficients β_i, the RKHS structure of the velocity fields is preserved - and thus the efficient computation of the gradients.

Algorithm



Experiments



10 examples images from the dataset of artificial characters used to evaluate our atlas estimation algorithm

We compare the original algorithm to the multiscale version using cross-validation.

- The dataset is randomly split into a training set (80%) and a test set (20%)
- Training: atlas estimation with the training images
- Test: registration of the estimated template image to the test images

The procedure is repeated 5 times and reproduced for different parameters (number of control points and kernel width).

σ_g	k_g	Original	Multiscale
		Atlas estimation	
5	36	86.0 ± 4.5	94.5 ± 0.8
4	49	83.6 ± 3.8	95.6 ± 0.4
3	100	81.6 ± 4.5	96.7 ± 0.5
		Registration	
5	36	90.2 ± 6.2	94.0 ± 6.3
4	49	89.3 ± 6.0	95.4 ± 4.2
3	100	86.6 ± 7.5	96.4 ± 3.8

Performance of the original and multiscale algorithms during training and test. Data: mean $\pm$ standard deviation of residuals percentage decrease. σ_g: kernel width; k_g: number of control points.

Estimation of the template image by the original and multiscale algorithms for $\sigma_g = 3$ and $k_g = 100$. Left: five estimated template images (for each fold of cross-validation). Right: first template image wrapped to the first five training images.

Conclusion

This atlas estimation algorithm is:

- **efficient**: we preserve the efficient optimization scheme of the original algorithm
- **simple**: we only add an outer layer of spatial regularization to the model
- **multiscale**: a coarse-to-fine strategy favors more realistic template images and promotes multiscale deformations.

References

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